Exercise 19

For the following exercises, determine whether there is a minimum or maximum value to each quadratic function. Find the value and the axis of symmetry.

$$f(x) = \frac{1}{2}x^2 + 3x + 1$$

Solution

Begin by factoring the coefficient of x^2.

$$f(x) = \frac{1}{2}(x^2 + 6x + 2)$$

In order to write this quadratic function in vertex form, it's necessary to complete the square, which makes use of the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = 6$, which means $B = 3$ and $B^2 = 9$. Add and subtract 9 on the right side within the parentheses and use the identity so that x appears in only one place.

$$\begin{aligned} f(x) &= \frac{1}{2}[(x^2 + 6x + 9) + 2 - 9] \\ &= \frac{1}{2}[(x + 3)^2 - 7] \\ &= \frac{1}{2}(x + 3)^2 - \frac{7}{2} \end{aligned}$$

Therefore, the vertex of the parabola is $\left(-3, -\frac{7}{2}\right)$. The axis of symmetry is $x = -3$, and the minimum (because the coefficient of x^2 is positive) is $y = -\frac{7}{2}$.